数理解析研究所講究録 1223 巻 2001 年 80-89

Stokes' theorem, self-adjointness of the Laplacian and Hodge's theorem for hyperbolic 3-cone-manifolds

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§1. Introduction

By a hyperbolic 3-cone-manifold, we will mean an orientable (not necessarily volume-finite) riemannian 3-manifold C of constant sectional curvature -1 with cone-type singularity along a 1-dimensional graph Σ which consists of geodesic segments in C. The subset $M := C - \Sigma$ has a smooth, incomplete hyperbolic structure whose metric completion is identical to the singular hyperbolic structure on C. The hyperbolic 3-manifold M is incomplete near Σ.

In this paper, we will inform that Stokes' theorem for smooth L^2-forms on the incomplete hyperbolic manifold M holds. The proof can be performed by following the argument described in Hodgson-Kerckhoff [5]. (In [5], Stokes' theorem in the case where each component of the singular locus Σ is homeomorphic to S^1 and the complement of an open tubular neighborhood of Σ is compact was shown.) Then from Stokes' theorem, by using a result of Gaffney [3], it is shown that there is a maximal extension of the Laplacian on M which is self-adjoint on its adequately defined domain. Thus, we have an extension of Hodge theory to hyperbolic 3-cone-manifolds whose singular loci are smooth 1-manifolds. Let E denote the flat vector bundle of local killing vector fields on the hyperbolic 3-manifold M. Then, if the singular locus Σ of the hyperbolic 3-cone-manifold C is a smooth 1-dimensional manifold, for any E-valued 1-form $\tilde{\omega}$ which represents an infinitesimal deformation of the hyperbolic structure on M around Σ and which satisfies some conditions related with the domain of the Laplacian ($\tilde{\omega}$ is called to be "in standard form"), there is a closed and co-closed E-valued 1-form ω which is equivalent to $\tilde{\omega}$ in the de Rham cohomology group $H^1(M; E)$. The 1-form ω is a representative with specific control on the asymptotic behavior near the singular locus.

§2. Stokes' theorem and self-adjointness of the Laplacian for hyperbolic 3-cone-manifolds

First we will give the definition of hyperbolic 3-cone-manifolds. Consider a smooth 3-dimensional manifold N, which has a path metric given by a gluing of the faces of finitely many geodesic polyhedra possibly with ideal verticies in the 3-dimensional hyperbolic space $\mathbf{H}^3$. The gluing is performed by orientation reversing isometries of $\mathbf{H}^3$. It is permitted that the polyhedra have "faces" on the sphere at infinity S^2_∞ which are not glued to another such "faces". We assume that the link of a vertex is piecewise linear homeomorphic to a sphere and the link of an ideal vertex is piecewise homeomorphic to a torus, an open annulus or an open disk. We also assume that the path metric on N is complete. The manifold N with the metric above is called a hyperbolic 3-cone-manifold.

The singular locus Σ of a hyperbolic 3-cone-manifold consists of the points with no neighborhood isometric to a ball in $\mathbf{H}^3$. It is a union of totally geodesic closed simplices of dimension 1. At each point of Σ in an open 1-simplex, there is a cone angle which is the sum of dihedral angles of polyhedra containing the point. The subset $N - \Sigma$ has a smooth riemannian metric of constant curvature -1, but this metric is incomplete near Σ if $\Sigma \neq \phi$.

Let C be a (not necessarily volume-finite) hyperbolic 3-cone-manifold with singular locus Σ. Let $M := C - \Sigma$ be a smooth (but incomplete) hyperbolic 3-manifold. A tubular neighborhood of a singular point of C, which is not a vertex, has the metric

$$dr^2 + \sinh^2 r d\theta^2 + \cosh^2 r dz^2,$$

by using the cylindrical coordinate. There are finitely many vertices of Σ.

We have a developing map of M from its universal covering space $\tilde{M}$,

$$\mathcal{D}_C : \tilde{M} \to \mathbf{H}^3,$$

and a holonomy representation,

$$\rho_C : \pi_1(M) \to \mathrm{PSL}_2(\mathbf{C}).$$

They are called a developing map and a holonomy representation of the cone-manifold C.

Let $\Omega^p(M)$ denote the space of smooth, real-valued p-forms of M and $\Omega^*(M)$ denote the space of smooth, real-valued forms on M. Let $\hat{d}$ be the usual exterior derivative of smooth real-valued forms on M:

$$\hat{d} \; : \; \Omega^p(M) \; \to \; \Omega^{p+1}(M).$$

Let $\hat{*}$ be the Hodge star operator defined by using the riemannian metric g on M:

$$g(\phi, \hat{*}\ \psi) dM = \phi \wedge \psi$$

for any real-valued p-form ϕ and $(3-p)$-form ψ. Let $\hat{\delta}$ be the adjoint of $\hat{d}$:

$$\hat{\delta} \,:\, \Omega^p(M) \,\rightarrow\, \Omega^{p-1}(M).$$

Let $\hat{\Delta}$ be the Laplacian on smooth real-valued forms for the riemannian manifold M:

$$\hat{\Delta} = \hat{d}\hat{\delta} + \hat{\delta}\hat{d}.$$

We will use $<\,,\,>$ to denote an L^2 inner product on real-valued forms:

$$<\xi,\eta>=\int_M \xi \wedge \hat{*}\,\eta = \int_M g(\xi,\eta\,)\;dM.$$

It is seen that Stokes' theorem for smooth L^2-forms on the incomplete hyperbolic manifold M can be proved as in Hodgson-Kerckhoff [5]. The proof is performed by using Cheeger's method in [1].

Theorem 1 (Stokes' theorem). *Let C be a hyperbolic 3-cone-manifold with singular locus Σ. Let $M := C - \Sigma$ be the smooth, incomplete hyperbolic 3-manifold. Then Stokes' theorem holds:*

$$\int_M \hat{d}\alpha \wedge \hat{*}\beta = \int_M \alpha \wedge \hat{*}\hat{\delta}\beta,$$

for smooth L^2-forms α,β on M such that $\hat{d}\alpha, \hat{\delta}\beta$ are L^2-forms on M.

If we define the domains of $\hat{d}$ and $\hat{\delta}$ by

$$\begin{aligned} \text{dom } \hat{d} \; &= \{\alpha \in \Omega^*(M)\;;\; \alpha \text{ and } \hat{d}\alpha \text{ are } L^2\}, \\ \text{dom } \hat{\delta} \; &= \{\beta \in \Omega^*(M)\;;\; \beta \text{ and } \hat{\delta}\beta \text{ are } L^2\}, \end{aligned}$$

then Theorem 1 saids that $<\hat{d}\alpha,\beta>=<\alpha,\hat{\delta}\beta>$ holds for all $\alpha \in \text{dom } \hat{d}$, $\beta \in \text{dom } \hat{\delta}$.

The strong closure $\overline{\hat{d}}$ of $\hat{d}$ is defined as follows (see [1]): $\overline{\hat{d}}\alpha = \eta$ means that α is an L^2-form and there exist $\alpha_i \in \text{dom } \hat{d}$ $(i \in \mathbf{N})$ such that $\alpha_i \rightarrow \alpha$, $\hat{d}\alpha_i \rightarrow \eta$. The domain of $\overline{\hat{d}}$ is defined by

$$\text{dom } \overline{\hat{d}} = \{\;\alpha\;;\;\alpha \text{ and } \overline{\hat{d}}\alpha \text{ are } L^2\text{-forms on } M\;\}.$$

In the same manner, the strong closure $\overline{\hat{\delta}}$ of $\hat{\delta}$ and its domain dom $\overline{\hat{\delta}}$ are defined.

The theorem above means that the manifold M has a negligible boundary (see [3],[4]). Then, by the result of Gaffney [3], for our manifold M, the Hilbert space closure $\overline{\hat{\Delta}}$ of $\hat{\Delta}$ is self-adjoint.

Theorem 2 (self-adjointness of $\overline{\hat{\Delta}}$). *Let C be a hyperbolic 3-cone-manifold with singular locus Σ. Let $M := C - \Sigma$ be the smooth, incomplete hyperbolic 3-manifold. Let $\overline{\hat{\Delta}}$ be the*

Hilbert space closure of the Laplacian for the riemannian manifold M so that

$$\text{the domain of } \overline{\hat{\Delta}} = \{\alpha \in \text{dom } \overline{\hat{d}} \cap \text{dom } \overline{\hat{\delta}}\ ;\ \overline{\hat{\delta}}\alpha \in \text{ dom } \overline{\hat{d}},\ \overline{\hat{d}}\alpha \in \text{ dom } \overline{\hat{\delta}}\}.$$

Then $\overline{\hat{\Delta}} = \overline{\hat{d}}\,\overline{\hat{\delta}} + \overline{\hat{\delta}}\,\overline{\hat{d}}$, and $\overline{\hat{\Delta}}$ is a closed, non-negative, self-adjoint and elliptic operator.

§3. Hodge theorem for hyperbolic 3-cone-manifolds

Let C be the hyperbolic 3-cone-manifold with singular locus Σ and $M = C - \Sigma$ be the hyperbolic 3-manifold considered in §2. Let G denote the group consisting of orientation preserving isometries of $\mathbf{H}^3$. The group G can be naturally identified with $\mathrm{PSL}_2(\mathbf{C})$. Let $\mathcal{G}$ denote the Lie algebra of G and Ad the adjoint representation of G on $\mathcal{G}$. Associated to the hyperbolic structure ρ_C is a flat $\mathcal{G}$ vector bundle E over M:

$$E = \widetilde{M} \times_{Ad \circ \rho_C} \mathcal{G}.$$

Let $\Omega^p(M; E)$ denote the space consisting of smooth E-valued p-forms on M. Let d be a covariant exterior derivative

$$d\ :\ \Omega^p(M; E)\ \rightarrow\ \Omega^{p+1}(M; E),$$

which is given by the flat connection on E. Then the pth de Rham cohomology group $H^p(M; E)$ of M with coefficients in E is defined by d.

There is a natural metric on E as follows. For each $x \in M$, the fiber E_x of the bundle E decomposes as a direct sum $\mathcal{P} \oplus \mathcal{K}$, where $\mathcal{P}$ consists of the infinitesimal pure translations at x and $\mathcal{K}$ consists of the infinitesimal rotations at x. Since an infinitesimal pure translation at x corresponds to a tangent vector to M at x, $\mathcal{P}$ is identified with the tangent space T_xM of M at x. Then we give $\mathcal{P}$ the metric induced from the riemannian metric on M. Similarly, since an element of $\mathcal{K}$ operates linearly and isometrically on the tangent space, a metric on $\mathcal{K}$ comes from identifying it with a subspace of $o(3)$ with its usual metric. In fact, $\mathcal{K}$ is identified with the total space $o(3)$. Then we give a metric on $\mathcal{P} \oplus \mathcal{K}$ by regarding the direct sum as an orthogonal direct sum. Let h denote the metric on E given as above.

Let $*$ denote the Hodge star operator on $\Omega^*(M; E)$ defined by using the riemannian metric h on E and the Hodge star operator $\hat{*}$ on $\Omega^*(M)$:

$$\alpha \wedge *\beta = (a\xi) \wedge (b\hat{*}\eta) = (ab)\ (\xi \wedge \hat{*}\eta) = h(a, b)\ g(\xi, \eta)\ dM,$$

for any $\alpha = a\xi$, $\beta = b\eta$ $(a, b \in \Omega^0(M; E), \xi, \eta \in \Omega^*(M))$. For two forms $\alpha = a\xi, \beta = b\eta \in \Omega^*(M; E)$, put

$$(\alpha, \beta) = \int_M \alpha \wedge *\beta = \int_M h(a, b)\ g(\xi, \eta) dM.$$

This is an L^2 inner product on $\Omega^*(M;E)$. We define

$$\delta \, : \, \Omega^p(M;E) \to \Omega^{p-1}(M;E)$$

by putting

$$\delta\alpha = (-1)^{3(p+1)+1} * d * \alpha$$

for any $\alpha \in \Omega^p(M;E)$. Then the associated Laplacian Δ is defined by

$$\Delta := d\delta + \delta d.$$

Let ∇ denote the Levi-Civita connection on E with respect to the metric h, and D denote a covariant exterior derivative induced by the connection ∇:

$$\begin{aligned} \nabla \quad &: \Omega^0(M;E) \to \Omega^1(M;E), \\ D \quad &: \Omega^p(M;E) \to \Omega^{p+1}(M;E). \end{aligned}$$

Put

$$D^*\alpha = (-1)^{3(p+1)+1} * D * \alpha,$$

for all $\alpha \in \Omega^p(M;E)$. Let $\{e_1, e_2, e_3\}$ be any orthonormal frame for TM and $\{\omega^1, \omega^2, \omega^3\}$ be the dual co-frame. Let $i()$ denote the interior product on forms. Then D and D^* are described as in the following:

$$\begin{aligned} D \quad &= \textstyle\sum_{j=1}^3 \omega^j \wedge \nabla_{e_j}, \\ D^* \quad &= -\textstyle\sum_{j=1}^3 i(e_j)\nabla_{e_j}. \end{aligned}$$

Put

$$\begin{aligned} T \quad &:= \textstyle\sum_{j=1}^3 \omega^j \wedge \ \mathrm{ad}(E_j), \\ T^* \quad &:= \textstyle\sum_{j=1}^3 i(e_j) \ \mathrm{ad}(E_j), \end{aligned}$$

where E_j is the element in the fiber over any point on M, which is the infinitesimal translation in the direction e_j at that point, and $\mathrm{ad}(E_j)$ sends an element Y in the fiber to $[E_j, Y]$. Then we have

$$\begin{aligned} d \quad &= D + T, \\ \delta \quad &= D^* + T^*. \end{aligned}$$

This shows a relationship between the flat structure on E, which is defined by the hyperbolic structure on M, and the natural metric h on E, which is defined by using the local geometry on M. (See Matsushima-Murakami [8] for the formulation above.)

As described above, at each point $x \in M$, the fiber E_x is decomposed into the orthogonal direct sum $\mathcal{P} \oplus \mathcal{K}$. Then the vector bundle E is decomposed into an orthogonal direct sum of two sub-bundles which we also denote as $\mathcal{P}$ and $\mathcal{K}$:

$$E = \mathcal{P} \oplus \mathcal{K}.$$

This decomposition induces a decomposition:

$$\Omega^p(M;E) = \Omega^p(M;\mathcal{P}) \oplus \Omega^p(M;\mathcal{K}).$$

The bundle $\mathcal{P}$ is naturally identified with the tangent bundle TM of M. The Levi-Civita connection ∇ restricted to $\mathcal{P}$-valued forms is the Levi-Civita connection on M. On $\mathcal{K} = o(3) \subset Hom(TM,TM)$, it is again the Levi-Civita connection induced by the one on TM. The operators D and D^* preserve the decomposition, while T and T^* map $\Omega^*(M;\mathcal{P})$ to $\Omega^*(M,\mathcal{K})$ and vice versa:

$$\begin{array}{ccccccc}
\Omega^*(M;\mathcal{P}) & \oplus & \Omega^*(M;\mathcal{K}) & \quad & \Omega^*(M;\mathcal{P}) & \oplus & \Omega^*(M;\mathcal{K}) \\
{\scriptstyle D,D^*}\big\downarrow & & \big\downarrow{\scriptstyle D,D^*} & & {\scriptstyle T,T^*}\big\downarrow & & \big\downarrow{\scriptstyle T,T^*} \\
\Omega^*(M;\mathcal{P}) & \oplus & \Omega^*(M;\mathcal{K}), & & \Omega^*(M;\mathcal{K}) & \oplus & \Omega^*(M;\mathcal{P}).
\end{array}$$

The Lie algebra $\mathcal{G} = sl_2(\mathbf{C})$ has a natural complex structure which is related to the decomposition $E = \mathcal{P} \oplus \mathcal{K}$ by $\mathcal{K} = i\ \mathcal{P}$. The multiplication by i in the Lie algebra induces a bundle isomorphism from $\mathcal{P}$ to $\mathcal{K}$, which respects the local geometry of M. For example, if t denotes an infinitesimal translation, then it is an infinitesimal rotation around the axis of t, and t and it are orthogonal. Now we will think of $\Omega^*(M;\mathcal{P})$ and $\Omega^*(M;\mathcal{K})$ as the real and imaginary parts of $\Omega^*(M;E)$:

$$\begin{array}{rcccc}
\Omega^*(M;E) & = & \mathrm{Re}\,\Omega^*(M;E) & \oplus & \mathrm{Im}\,\Omega^*(M;E) \\
& = & \Omega^*(M;\mathcal{P}) & \oplus & \Omega^*(M;\mathcal{K}) \\
& = & \Omega^*(M;\mathcal{P}) & \oplus & i\,\Omega^*(M;\mathcal{P}).
\end{array}$$

An E-valued p-form α is a pair of a real part α_{real} and a imaginary part α_{imag}. The real part α_{real} is a $\mathcal{P}$-valued p-form on M. If v is a $\mathcal{P}$-valued 0-form (namely a tangent vector field) on M, then $(dv)_{real}$ is $Dv \in \Omega^1(M;\mathcal{P})$ $(= \Omega^1(M;TM) = Hom(TM,TM))$, which is also equal to ∇v, and $(dv)_{imag}$ is $Tv \in \Omega^1(M;\mathcal{K})$ $(= i\ \Omega^1(M;\mathcal{P}) = i\ \Omega^1(M;TM)$ $= i\ Hom(TM,TM))$. By using the orthonormal frame $\{e_k, e_l, e_j\}$ and the dual co-frame $\{\omega^k, \omega^l, \omega^j\}$, we can describe a canonical isomorphism between skew-symmetric elements of $Hom(TM,TM)$ and vector fields:

$$Hom(TM,TM)_{skew} \ni e_l \otimes \omega^j - e_j \otimes \omega^l \to e_k \in \Omega^0(M;TM).$$

If v is a tangent vector field on M, Dv is an element of $Hom(TM, TM)$. The skew-symmetric part $(Dv)_{skew}$ of Dv is called the curl of v, and is denoted by $curl\ v$. By the isomorphism above, $curl\ v$ is regarded as a vector field on M. Note that this vector field is the half of the usual curl considered in elementary vector calculus. The trace of Dv is called the divergence of v, and is denoted by $div\ v$. The traceless, symmetric part of Dv is called the strain of v, and is denoted by $str\ v$.

If v is a locally defined tangent vector field on M, then we can consider a local section of the bundle E, which is defined by $s_v = v - i\ curl\ v$. Call it the canonical lift of v.

Let σ be any closed smooth E-valued 1-form on M. Choosing a point $x \in M$, we can locally define a section $\int_x \sigma$ of the bundle E by integrating σ along paths beginning at x, which is called the associated local section. Note that we are using the flat connection on E to identify the fibers at different points along the path in order to do the integration. Since σ is closed, the value of the integral depends only on the homotopy class of the path; a well-defined section is determined on any simply connected subset of M. Then $d \int_x \sigma = \sigma$ on such a subset. In general, the section will not extend to a global section on M.

In the rest of the paper, we assume that the singular locus Σ of the cone-manifold C is a smooth 1-manifold:

$$\Sigma \approx \mathbf{R} \sqcup \ldots \sqcup \mathbf{R} \sqcup S^1 \sqcup \ldots \sqcup S^1.$$

Some examples of hyperbolic 3-cone-manifolds with infinite volume, whose singular loci are homeomorphic to $\mathbf{R}$, are illustrated in [9].

In a tubular neighborhood U_k of each component Σ_k of Σ, we use cylindrical coordinates, (r, θ, z). Then the hyperbolic metric on U_k is $dr^2 + \sinh^2 r d\theta^2 + \cosh^2 r dz^2$. We will use the orthonormal frame $\{e_1, e_2, e_3\}$ of TM adapted to this coordinate system:

$$e_1 := \frac{\partial}{\partial r},\ e_2 := \frac{1}{\sinh r}\frac{\partial}{\partial \theta},\ e_3 := \frac{1}{\cosh r}\frac{\partial}{\partial z}.$$

Then the dual co-frame $\{\omega^1, \omega^2, \omega^3\}$ is

$$\omega^1 = dr,\ \omega^2 = \sinh r\ d\theta,\ \omega^3 = \cosh r\ dz.$$

An E-valued 1-form can be interpretted as a complex-valued section of $\mathcal{P} \otimes T^*M \cong TM \otimes T^*M \cong Hom(TM, TM)$. Then an E-valued 1-form can be described as a matrix in $M_3(\mathbf{C})$ whose (i, j) entry is the coefficient of $e_i \otimes \omega^j$.

The form in (1) below is a closed and co-closed form which represents an infinitesimal deformation which does not change the real part of the complex length of an element of the fundamental group of U_k which is so called the meridian of U_k. The meridian is the class of the fundamental group which wraps around Σ_k once and bounds a singular disk with cone angle equal to that of Σ_k. The infinitesimal deformation preserves the property that the meridian is elliptic. Then it gives a small deformation of the cone-manifold U_k to a cone-manifold. The infinitesimal deformation also has the remarkable property that it decreases the cone angle.

$$\tilde{\omega}_{(1)} = \begin{pmatrix} \frac{-1}{\cosh^2 r \ \sinh^2 r} & 0 & 0 \\ 0 & \frac{1}{\sinh^2 r} & \frac{-i}{\cosh r \ \sinh r} \\ 0 & \frac{-i}{\cosh r \ \sinh r} & \frac{-1}{\cosh^2 r} \end{pmatrix} \tag{1}$$

The form in (2) below is a closed and co-closed form which represents an infinitesimal deformation which leaves the holonomy of the meridian (hence the cone angle) unchanged. If Σ_k is homeomorphic to S^1, this deformation stretches the length of Σ_k.

$$\tilde{\omega}_{(2)} = \begin{pmatrix} \frac{-1}{\cosh^2 r} & 0 & 0 \\ 0 & -1 & \frac{-i \ \sinh r}{\cosh r} \\ 0 & \frac{-i \ \sinh r}{\cosh r} & \frac{\cosh^2 r+1}{\cosh^2 r} \end{pmatrix} \tag{2}$$

Definition (in standard form). Let $\tilde{\omega}$ be a smooth, closed, E-valued 1-form on M such that $\delta\tilde{\omega}, d(\delta\tilde{\omega}), \delta d(\delta\tilde{\omega})$ are L^2. We say that the 1-form $\tilde{\omega}$ is in standard form if the following conditions are satisfied:

- The associated local section $\int_x \tilde{\omega}$ is the canonical lift of its real part:
$$\int_x \tilde{\omega} = \left(\int_x \tilde{\omega}\right)_{real} - i \ curl \ \left(\int_x \tilde{\omega}\right)_{real}, \text{ for any } x \in M.$$
- In a tubular neighborhood U_k of a component Σ_k of the singular locus Σ,
$$\tilde{\omega} = h_1 \, \tilde{\omega}_{(1)} + h_2 \, \tilde{\omega}_{(2)} \ \text{ for some } h_1, h_2 \in \mathbf{C}.$$

Theorem 3 (Hodge theorem for hyperbolic 3-cone-manifolds). *Let C be a hyperbolic 3-cone-manifold with singular locus Σ. Let $M := C - \Sigma$ be the smooth, incomplete hyperbolic 3-manifold. Assume that Σ is a disjoint union of smooth 1-manifolds; $\Sigma \approx \mathbf{R} \sqcup \ldots \sqcup \mathbf{R} \sqcup S^1 \sqcup \ldots \sqcup S^1$. Let $\tilde{\omega} \in \Omega^1(M; E)$ be a smooth, E-valued 1-form which is in standard form. Then there exists a smooth, closed and co-closed E-valued 1-form ω, which is cohomologous to $\tilde{\omega}$ and whose associated local section $\int_x \omega$ is the canonical lift of a divergence-free, harmonic vector field. Moreover, there is a unique such form satisfying the condition that $\tilde{\omega} - \omega = ds$ where s is a globally defined L^2 section of E.*

Outline of the proof. We want to solve the equation $\Delta s = \delta\tilde{\omega}$ for a globally defined section s of E. Since the associated local section $\int_x \tilde{\omega}$ is the canonical lift of its real part, $\delta\tilde{\omega}$ is also the canonical lift of its real part. Thus, it suffices to solve $\Delta v = (\delta\tilde{\omega})_{real}$ for a globally defined vector field v on M. Let $\zeta \in \Omega^1(M)$ be a smooth, real-valued 1-form which is the dual to the vector field $(\delta\tilde{\omega})_{real}$. Then, by using a Weitzenböck formula, we can see that it suffices to solve

$$(\hat{\Delta} + 4)\tau = \zeta,$$

for a smooth, real-valued 1-form $\tau \in \Omega^1(M)$. Now we apply the self-adjointness of the closure $\overline{\hat{\Delta}}$ of the Laplacian $\hat{\Delta}$ on $\Omega^*(M)$. Since ζ is in the domain of $\overline{\hat{\Delta} + 4}$, then by Theorem 2, there is a unique solution $\tau \in$ the domain of $\overline{\hat{\Delta} + 4}$. Since ζ is smooth, then, by the usually regularity theory for elliptic operators, τ is also smooth. Therefore, we can find a globally defined smooth section s of E which satisfies $\Delta s = \delta\tilde{\omega}$. Then put $\omega := \tilde{\omega} - ds$. It is easy to see that ω and s satisfy the condition described in the theorem. □

If each component Σ_k of the singular locus Σ is homeomorphic to S^1 and $M - \sqcup_k U_k$ is compact, each cohomology class has a representative in standard form (see Lemma 3.3 in [5]).

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